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A HYBRID APPROACH OF TOPSIS AND FUZZY GOAL PROGRAMMING FOR BI-LEVEL MODM PROBLEMS WITH FUZZY PARAMETERS

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ABSTRACT

In this paper, TOPSIS (technique for order preference by similarity to ideal solution) approach for solving bi-level multi-objective programming problems (BL-MOPP) with fuzzy parameters is proposed. These fuzzy parameters are assumed to be characterized as fuzzy numbers, reflecting the experts' imprecise or fuzzy understanding of the nature of parameters in the problem formulation process. Using the level sets of fuzzy parameters, the corresponding non fuzzy bi-level programming problem is introduced. The proposed approach for obtaining the satisfactory solution of the BL-MOPP with fuzzy parameters includes the membership functions of the distance function from the positive ideal solution (PIS), the membership functions of the distance function from the negative ideal solution (NIS) and the membership functions of the upper level decision variables vector with possible tolerances. Also, a modified TOPSIS approach is presented in this paper. Illustrative numerical example is given to demonstrate the proposed TOPSIS and modified TOPSIS approach. Also, a comparison between the proposed TOPSIS and the modified TOPSIS approaches with existing Algorithms is given to clarify the powerful of the proposed approaches.

Keywords: *Bi-level programming; Fuzzy sets; Fuzzy parameters; TOPSIS; Fuzzy goal programming; multi-objective programming.*

1. INTRODUCTION

Bi-level mathematical programming (BLMP) is identified as mathematical programming that solves decentralized planning problems with two decision makers (DMs) in two-levels or hierarchical organization [12, 26]. The basic concept of BLMP is that a first-level decision maker (FLDM) (the leader) sets his goals and/or decisions and then asks each subordinate level of the organization for their optima which are calculated in isolation; the second-level decision maker (SLDM) (the follower) decisions are then submitted and modified by the FLDM with consideration of the overall benefit of for the organization; the process is continued until a satisfactory solution is reached [12].

A bibliography of the related references on bi-level programming in both linear and non-linear cases, which is updated biannually, can be found in [32]. The use of the fuzzy set theory [34] for decision problems with several conflicting objectives was first introduced by Zimmermann [35] Thereafter, various versions of fuzzy programming (FP) have been investigated and widely circulated in literature [6, 26, 27, 28].

When formulating a mathematical programming problem which closely describes and represents the real-world decision situation, various factors of the real-world system should be reflected in the description of objective functions and constraints. Naturally, these objective functions and constraints involve many parameters whose possible values may be assigned by the experts [31, 33, 38]. It has been observed that, in most real-world situations, for example, power markets and business management, the possible values of these parameters are often only imprecisely or ambiguously known to the experts and cannot be described by precise values. With this observation, it would be certainly more appropriate to interpret the experts understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy sets [33, 38].

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Technique for order performance by similarity to ideal solution (TOPSIS), one of the known classical multiple criteria decisionmaking (MCDM) method, bases upon the concept that the chosen alternative should have the shortest distance from thepositive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). It was first developed by Hwang and Yoon[17] for solving a multiple attribute decision making problem. A similar concept has also been pointed out by Zeleny[39]. Lia *et al.* [20] extended the concept of TOPSIS to develop a methodology for solving multiple objective decision making (MODM) problems. Recently, Abo-Sinna [4] extended TOPSIS approach to solve multi-objective dynamics programming (MODP) problems.

As he showed that using the fuzzy max–min operator with non-linear membership functions, the obtained solutions are ways non-dominated by the original MODP problems. Further extension of TOPSIS for large scale multi-objective non-linear programming problems with block angular structure was presented by Abo-Sinna *et al.* in [3,5]. Deng *et al.* [16] formulated the inter-company comparison process as a multi-criteria analysis model, and presented an effective approach by modifying TOPSIS for solving such a problem. Chen [15] extended the concept of TOPSIS to develop a methodology for solving multi-personmulti-criteria decision-making problems in a fuzzy environment and he defined the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS).

Generally, TOPSIS provides a broader principle of compromise for solving multiple criteria decision making problems. It transfers m-objectives (criteria), which are conflicting and non-commensurable, into two objectives (the shortest distance from the PIS and the longest distance from the NIS). They are commensurable and most of time conflicting.Then, the bi-objective problem can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS's compromise solution by a second-order compromise. The max–min operator is then considered as a suitable one to resolve the conflict between the new criteria (the shortest distance from the PIS and the longest distance from the NIS) [3, 5, 18].

Thus, contrary to what is stated in the first paragraph Pramanik and Dey [24] proposed a fuzzy goal programming model for solving BL-MOPP that doesn't follow the basic concepts of bi-level programming and neglect the upper level decision variable vectors. As this is equivalent to solve the BL-MOPP as a single level multi-objective programming problem. Hence, bi-level programming is a hierarchical optimization problem consisting of two levels, the first of which (the leader's level) is dominant over the other (the follower's one). The order of the play is very important, the choice of the dominant level limits or highly affects the choice or strategy of the lower level. Knowing the selection of the leader, the FGP model of BL-MOPP in which the decision variables of the FLDM appear as a membership functions is solved to obtain a satisfactory solution.

By considering the basic concept of bi-level programming problem, the TOPSIS approach for bi-level MODM of IbrahimA. Baky[8] is extended to solve bi-level multi-objective programming problem with fuzzy parameters. To formulate the model of BL-MOPP with fuzzy parameters, for a prescribed value ofα, the model is converted toa deterministic BL-MOPP using the level set of fuzzy numbers. Also, a modified TOPSIS approach in which the bi-objective problem is solved by a fuzzy goal programming (FGP) is presented. The remainder of this paper is organized as follows. Section 2 presents some preliminaries. Section 3 and section 4 briefly discuss problem formulation and model formulation of BL-MOPP with fuzzy parameters. The proposed fuzzy TOPSIS approach is developed in section 5 for BL-MOPP with fuzzy parameters and Section 6 presents the algorithm of the TOPSIS approach for solving BL-MOPP with fuzzy parameters. A modified TOPSIS approach is presented in section 7. The following section presents an illustrative numerical example to demonstrate the proposed TOPSIS and modified TOPSIS approaches. Also, a comparison between the proposed TOPSIS and the modified TOPSIS approaches with existing Algorithms is given to clarify the powerful of the proposed approaches. The concluding remarks are made in section 9.

2. PRELIMINARIES

In this section, some basic concepts of fuzzy set theory and distance measures have been introduced, for more details see [38] for fuzzy set theory and [3-5, 37] for distance measures.

Definition 2.1: Let R be the space of real numbers. A Fuzzy set $\tilde{A}_i$ is a set of ordered pairs$\left\{\left(x, \mu_{\tilde{A}_i}(x)\right) \middle| x \in R\right\}$, where $\mu_{\tilde{A}_i}(x): \rightarrow [0,1]$ is called membership function of fuzzy set.

Definition 2.2: A convex fuzzy set, $\tilde{A}_i$, is a fuzzy set in which: $\forall$ x, y $\in$R,$\forall\lambda \in$ [0, 1] $\mu_{\tilde{A}_i}(\lambda x + (1-\lambda)y \geq \min\left[\mu_{\tilde{A}_i}(x), \mu_{\tilde{A}_i}(y)\right]$.

Definition2.3: A fuzzy set Ã is called positive if its membership function is such that $\mu_{\tilde{A}_i}(x) = 0, \forall x \leq 0$

Definition 2.4: Triangular fuzzy number (TFN) is a convex fuzzy set which is defined as Ã= $(x, \mu_{\tilde{A}_i}(x))$ where:

$$\mu_{\tilde{A}_i}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ \frac{(c-x)}{(c-b)} & b \le x \le c \\ 0 & \text{other wise} \end{cases} \quad (2.1)$$

For convenience, TFN represented by three real parameters (a, b, c) which are $(a \le b \le c)$ will be denoted by the triangle a, b, c (Fig.1).

Definition 2.5: The α–level set of a fuzzy set Ã is a non-fuzzy set denoted by $(\tilde{A})_\alpha$ for which the degree of it's membership functions exceed or equal to a real number α ∈ [0, 1], i.e.$(\tilde{A})_\alpha = \{x | \mu_{\tilde{A}}(x) \ge \alpha\}$.

The α-level set of $\tilde{a}$ is then; $\tilde{a}_\alpha = [\, \tilde{a}_\alpha^L, \tilde{a}_\alpha^U \,]$ that is $\tilde{a}_\alpha^L = (1-\alpha)a + \alpha b$, and $\tilde{a}_\alpha^U = (1-\alpha)c + \alpha b$, where, $\tilde{a}_\alpha^L$ and $\tilde{a}_\alpha^U$ represent the lower and upper cuts respectively, shown in (Fig.1).

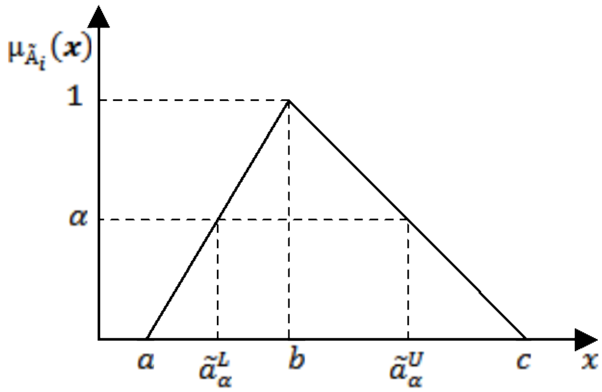

Fig.1: Triangular Fuzzy Number

Definition 2.6: Intersection of two fuzzy sets $\tilde{A}_1$and $\tilde{A}_2$ with membership functions $\mu_{\tilde{A}_1}(x)$ and $\mu_{\tilde{A}_2}(x)$ respectively is defined by a fuzzy set $\tilde{A}_3$ whose membership function is defined by $\mu_{\tilde{A}_3}(x) = \mu_{\tilde{A}_1 \cap \tilde{A}_2}(x) = \min\left(\mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x)\right)$, $x \in R$.

Consider the vector of objective functions$F(\boldsymbol{x}) = \left(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x})\right)$ to be minimizedand ideal vector of objective functions $F^* = (f_1^*, f_2^*, \ldots, f_m^*)$(ideal point- reference point- positive ideal solution (PIS)) in the m-objective space. And consider the vector of anti-ideal solution of objective functions $F^- = (f_1^-, f_2^-, \ldots, f_m^-)$(anti-ideal point – nadir point – negative ideal $solution$ (NIS)). Where$f_j^* = \underbrace{min}_{x \in G} f_j(\boldsymbol{x})andf_j^- = \underbrace{max}_{x \in G} f_j(\boldsymbol{x})$, $j = 1,2, \ldots, m$. And G is a convex constraints feasible set. As the measure of ''closeness'',L_p-metric is used. The L_p-metric defines the distance between two points $F(\boldsymbol{x})$and F^*. If the objective functions $f_j(\boldsymbol{x})$, $j = 1,2, \ldots, m$, are not expressed in commensurable units, then a scaling function for every objective function, usually, this dimensionless is the interval [0, 1]. In this case, the following metric could be used:

$$d_p = \left\{ \sum_{j=1}^{m} w_j^p \left[\frac{f_j^* - f_j(x)}{f_j^* - f_j^-} \right]^p \right\}^{\frac{1}{p}}, \; p = 1,2, \ldots, \infty. \quad (2.2)$$

where w_j , $j = 1,2, \ldots, m$,are the relative importance (weights) of objectives.

3. PROBLEM FORMULATION

Assume that there are two levels in a hierarchy structure with first-level decision maker (FLDM) and second-level decision maker (SLDM). Let the vector of decision variables $x = (x_1, \; x_2) \in R^n$ be partitioned between the two planners. The first-level decision maker has control over the vector $x_1 \in R^{n_1}$and the second-level decision maker has control over the vector$x_2 \in R^{n_2}$, where $n = n_1 + n_2$. Further more, assumethat

$$\widetilde{F}_i(x_1, x_2): \; R^{n_1} \times R^{n_2} \to R^{m_i}, \quad i = 1,2 \quad (3.1)$$

are the first-level and the second-level vector objective functions, respectively. So the BL-MOPP with fuzzy parameters of minimization type may be formulated as follows [9, 24, 31]:

[1st Level]

$$\underbrace{\text{Min}}_{x_1} \tilde{F}_1(x_1,x_2) = \underbrace{\text{Min}}_{x_1} \left(\tilde{f}_{11}(x_1,x_2), \tilde{f}_{12}(x_1,x_2), \dots, \tilde{f}_{1m_1}(x_1,x_2)\right), \tag{3.2}$$

where x_2 solves

[2nd Level]

$$\underbrace{\text{Min}}_{x_2} \tilde{F}_2(x_1,x_2) = \underbrace{\text{Min}}_{x_2} \left(\tilde{f}_{21}(x_1,x_2), \tilde{f}_{22}(x_1,x_2), \dots, \tilde{f}_{2m_2}(x_1,x_2)\right), \tag{3.3}$$

subject to

$$x \in G = \left\{x = (x_1,x_2) \in R^n \middle| \tilde{A}_1 x_1 + \tilde{A}_2 x_2 \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, x \geq 0, b \in R^m \right\} \neq \varphi, \tag{3.4}$$

where

$$\tilde{f}_{ij}(x) = \tilde{c}_1^{ij} x_1 + \tilde{c}_2^{ij} x_2, \quad i = 1,2 \quad j = 1,2, \dots, m_i,$$
$$\tilde{f}_{ij}(x) = \tilde{c}_{11}^{ij} x_{11} + \tilde{c}_{12}^{ij} x_{12} + \cdots + \tilde{c}_{1n_1}^{ij} x_{1n_1} + \tilde{c}_{21}^{ij} x_{21} + \tilde{c}_{22}^{ij} x_{22} + \cdots + \tilde{c}_{2n_2}^{ij} x_{2n_2}, \tag{3.5}$$

and where m_i, $i = 1,2$ are the number of DMj's objective functions, m is the number of constraints, $\tilde{c}_k^{ij} = \left(\tilde{c}_{k1}^{ij}, \tilde{c}_{k2}^{ij}, \dots, \tilde{c}_{kn_k}^{ij}\right)$, $k = 1,2$ and $\tilde{c}_{kn_k}^{ij}$ are constants, $\tilde{A}_i$ are the coefficient matrices of size$m \times n_i$,$i = 1,2$., the control variables $x_1 = \left(x_1^1, x_1^2, \dots, x_1^{n_1}\right)$ and $x_2 = \left(x_2^1, x_2^2, \dots, x_2^{n_2}\right)$, and G is the bi-level convex constraints feasible choice set.

4. MODEL FORMULATION OF BL-MOPP WITH FUZZY PARAMETERS

The individual optimal solution of FLDM and SLDM objective functions would be considered when scaling every objective function. Then for a prescribed value of α, minimization-type objective function [24, 30, 31], $\tilde{f}_{ij}(x)(i = 1,2)$, $(j = 1,2, \dots, m_i)$ can be replaced by the lower bound of its α-level, i.e.

$$\left(\tilde{f}_{ij}(x)\right)_\alpha^L = \left(\tilde{c}_1^{ij}\right)_\alpha^L x_1 + \left(\tilde{c}_2^{ij}\right)_\alpha^L x_2 \ (i = 1,2), \ (j = 1,2, \dots, m_i) \tag{4.1}$$

Similarly, for a prescribed value ofα, maximization-type objective function, $\tilde{f}_{ij}(x)$, $i = 1,2$, $j = 1,2, \dots, m_i$can be replaced by the upper bound of its α-level (*α-cut)*, i.e.

$$\left(\tilde{f}_{ij}(x)\right)_\alpha^U = \left(\tilde{c}_1^{ij}\right)_\alpha^U x_1 + \left(\tilde{c}_2^{ij}\right)_\alpha^U x_2, \ (i = 1,2), (j = 1,2, \dots, m_i) \tag{4.2}$$

For the inequality constraints,

$$\sum_{j=1}^n \tilde{A}_{ij} x_j \geq \tilde{b}_i (i = 1,2, \dots, r_1)$$

And $$\sum_{j=1}^n \tilde{A}_{ij} x_j \leq \tilde{b}_i (i = 1,2, \dots, r_2) \tag{4.3}$$

Can be rewritten by the following constraints as [16, 21, 22, 28]:

$$\sum_{j=1}^n \left(\tilde{A}_{ij}\right)_\alpha^U x_j \geq \left(\tilde{b}_i\right)_\alpha^L (i = 1,2, \dots, r_1)$$

And $$\sum_{j=1}^n \left(\tilde{A}_{ij}\right)_\alpha^L x_j \leq \left(\tilde{b}_i\right)_\alpha^U (i = r_1 + 1, \dots, r_2) \tag{4.4}$$

For equality constraints;

$$\sum_{j=1}^n \tilde{A}_{ij} x_j = \tilde{b}_i (i = r_2 + 1, \dots, m) \tag{4.5}$$

Can be replaced by two equivalent constraints;

$$\sum_{j=1}^n \left(\tilde{A}_{ij}\right)_\alpha^L x_j \leq \left(\tilde{b}_i\right)_\alpha^U (i = r_2 + 1, \dots, m)$$
$$\sum_{j=1}^n \left(\tilde{A}_{ij}\right)_\alpha^U x_j \geq \left(\tilde{b}_i\right)_\alpha^L (i = r_2 + 1, \dots, m) \tag{4.6}$$

For proof of equivalency of the above equations (4.5) and (4.6), see Lee and Li [21, 31,38]. Therefore, for a prescribed value of α, the minimization-type problem reduces to the following BL-MOPP:

[1st Level]

$$\underbrace{\text{Min}}_{x_1} \left(\tilde{F}_1(x)\right)_\alpha^L = \underbrace{\text{Min}}_{x_1} \left(\left(\tilde{f}_{11}(x)\right)_\alpha^L, \left(\tilde{f}_{12}(x)\right)_\alpha^L, \dots, \left(\tilde{f}_{1m_1}(x)\right)_\alpha^L\right) \tag{4.7}$$

where x_2 solves

[2nd Level]

$$\underbrace{\text{Min}}_{x_2}\left(\tilde{F}_2(x)\right)_\alpha^L = \underbrace{\text{Min}}_{x_2}\left(\left(\tilde{f}_{21}(x)\right)_\alpha^L, \left(\tilde{f}_{22}(x)\right)_\alpha^L, \dots, \left(\tilde{f}_{2m_2}(x)\right)_\alpha^L\right) \tag{4.8}$$

subject to

$$\sum_{j=1}^{n}\left(\tilde{A}_{ij}\right)_\alpha^U x_j \geq \left(\tilde{b}_i\right)_\alpha^L (i = 1,2,\dots,r_1, r_2+1,\dots,m)$$
$$\sum_{j=1}^{n}\left(\tilde{A}_{ij}\right)_\alpha^L x_j \leq \left(\tilde{b}_i\right)_\alpha^U (i = r_1+1,\dots,r_2, r_2+1,\dots,m) x_j \geq 0, (j = 1,2) \tag{4.9}$$

Thus, for a prescribed α, the BL-MOPP with fuzzy parameters reduces to deterministic BL-MOPP which can be solved by using TOPSIS approach proposed in [8] by Baky.

5. TOPSIS APPROACH FOR BL-MOPP WITH FUZZY PARAMETERS

In most practical situations, we might like to have a decision, which not only makes as much profit as possible, but also avoids as much risk as possible. This concept has been developed by Hwang and Yoon [17]. They provided a new approach, TOPSIS, for solving a multiple attribute decision-making (MADM) problems. It is based upon the principle that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). Hwang and Yoon used both PIS (F^*) and NIS (F^-) to normalize the distance family and obtain the form of distancefamily of Eq. (2.2). Lia *et al.* [18] extended the concept of TOPSIS to develop a methodology for solving multiple objective decision making (MODM) problems. In this paper, the researchers further extended the concept of TOPSIS [18] for BL-MOPP with fuzzy parameters.

5.1. The TOPSIS approach for the first level MOP Problem

Consider the first level of minimization type problem of the BL-MOPP with fuzzy parameters (3.2):

$$\underbrace{\text{Min}}_{x_1}\tilde{F}_1(x_1,x_2) = \underbrace{\text{Min}}_{x_1}\left(\tilde{f}_{11}(x_1,x_2), \tilde{f}_{12}(x_1,x_2), \dots, \tilde{f}_{1m_1}(x_1,x_2)\right),$$

subject to

$$x \in G = \left\{x = (x_1,x_2) \in R^n \middle| \tilde{A}_1x_1 + \tilde{A}_2x_2 \begin{pmatrix}\leq\\=\\\geq\end{pmatrix} b, x \geq 0, b \in R^m\right\} \neq \varphi, \tag{5.1}$$

Thus, for a specified value of α, the first level is converted to deterministic one. Then the TOPSIS approach of Lia et al. [18] that solves single-level MODM problems is considered, in this paper, to solve the first level MOP problem with fuzzy parameters .The TOPSIS model formulation of this approach can be briefly stated as following, for more details see [18]:

$$\text{Min } d_P^{PIS^u}(x)$$
$$\text{Max} d_P^{NIS^u}(x)$$

subject to

$$x \in G = \left\{x = (x_1,x_2) \in R^n \middle| \tilde{A}_1x_1 + \tilde{A}_2x_2 \begin{pmatrix}\leq\\=\\\geq\end{pmatrix} b, x \geq 0, b \in R^m\right\} \neq \varphi, \tag{5.2}$$

where

$$d_P^{PIS^u}(x) = \left\{\sum_{j=1}^{m_1} w_j^p \left[\frac{\tilde{f}_{1j}(x) - f_{1j}^*}{f_{1j}^- - f_{1j}^*}\right]^p\right\}^{\frac{1}{p}} \text{and} d_P^{NIS^u}(x) = \left\{\sum_{j=1}^{m_1} w_j^p \left[\frac{f_{1j}^- - \tilde{f}_{1j}(x)}{f_{1j}^- - f_{1j}^*}\right]^p\right\}^{\frac{1}{p}} \tag{5.3}$$

and where $f_{1j}^* = \underbrace{min}_{x\in G}\tilde{f}_{1j}(x) = \underbrace{min}_{x\in G}\left(\tilde{f}_{1j}(x)\right)_\alpha^L$, is the individual positive ideal solutions, $f_{1j}^- = \underbrace{max}_{x\in G}\tilde{f}_{1j}(x) = \underbrace{max}_{x\in G}\left(\tilde{f}_{1j}(x)\right)_\alpha^U$, is the individual negative ideal solutions and w_j, is the relative importance (weights) of objectives. As the problem of minimization type then $\tilde{f}_{1j}(x) = \left(\tilde{f}_{1j}(x)\right)_\alpha^L$. Let $F^{u^*} = \left(f_{11}^*, f_{12}^*, \dots, f_{1m_1}^*\right)$ and $F^{u^-} = \left(f_{11}^-, f_{12}^-, \dots, f_{1m_1}^-\right)$. Assume that the membership functions ($\mu_1(x)$ and $\mu_2(x)$) of the two objective functions are linear between $\left(d_p^u\right)^*$ and $\left(d_p^u\right)^-$ which are:

$$\left(d_P^{PIS^u}\right)^* = \underbrace{min}_{x\in G} d_P^{PIS^u}(x) \text{and the solution is } x^P, \tag{5.4}$$

$$\left(d_P^{NIS^u}\right)^* = \underbrace{max}_{x\in G} d_P^{NIS^u}(x) \text{and the solution is } x^N, \tag{5.5}$$

$$\left(d_P^{PIS^u}\right)^- = d_P^{PIS^u}(x^N) \text{and} \left(d_P^{NIS^u}\right)^- = d_P^{NIS^u}(x^P). \tag{5.6}$$

Also, as proposed in [8] by Baky that $\left(d_P^{PIS^u}\right)^-$ and $\left(d_P^{NIS^u}\right)^-$ can be taken as $\left(d_P^{PIS^u}\right)^- = \underbrace{max}_{x\in G} d_P^{PIS^u}(x)$ and $\left(d_P^{NIS^u}\right)^- = \underbrace{min}_{x\in G} d_P^{NIS^u}(x)$, respectively. Let $d_P^{u^*} = \left(\left(d_P^{PIS^u}\right)^*, \left(d_P^{NIS^u}\right)^*\right)$ and $d_P^{u^-} = \left(\left(d_P^{PIS^u}\right)^-, \left(d_P^{NIS^u}\right)^-\right)$. Thus $\mu_1(x) \equiv \mu_{d_P^{PIS^u}}(x)$ and $\mu_2(x) \equiv \mu_{d_P^{NIS^u}}(x)$ can be obtained as (see: Fig. 2):

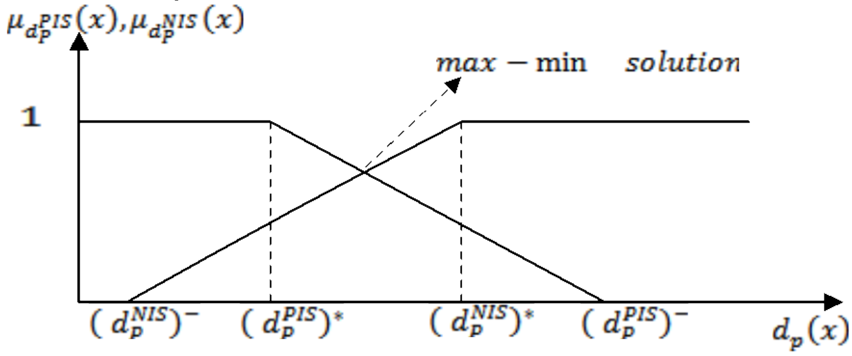

Fig.2: The membership functions of $\mu_{d_P^{PIS}}(x)$ and $\mu_{d_P^{NIS}}(x)$

Applying the max-min decision model, which is proposed by Bellman and Zadeh[13] and extended by Zimmermann [35]. We can resolve (5.2) and obtaining the satisfying decision of first level MOP problem, $x^{u^*} = \left(x_1^{u^*}, x_2^{u^*}\right)$, by solving the following problem:

$$\mu_D(x) = \underbrace{max}_{x\in G}\left\{min\left(\mu_1(x), \mu_2(x)\right)\right\}, \tag{5.7}$$

$$\mu_1(x) = \begin{cases} 1 & \text{if} d_P^{PIS^u}(x) < \left(d_P^{PIS^u}\right)^* \\ 1 - \frac{d_P^{PIS^u}(x) - \left(d_P^{PIS^u}\right)^*}{\left(d_P^{PIS^u}\right)^- - \left(d_P^{PIS^u}\right)^*} & \text{if } \left(d_P^{PIS^u}\right)^* \le d_P^{PIS^u}(x) \le \left(d_P^{PIS^u}\right)^- \\ 0 & \text{if}\left(d_P^{PIS^u}\right)^- < d_P^{PIS^u}(x) \end{cases} \tag{5.8}$$

$$\mu_2(x) = \begin{cases} 1 & \text{if} d_P^{NIS^u}(x) \ge \left(d_P^{NIS^u}\right)^* \\ 1 - \frac{\left(d_P^{NIS^u}\right)^* - d_P^{NIS^u}(x)}{\left(d_P^{NIS^u}\right)^* - \left(d_P^{NIS^u}\right)^-} & \text{if } \left(d_P^{NIS^u}\right)^- \le d_P^{NIS^u}(x) \le \left(d_P^{NIS^u}\right)^* \\ 0 & \text{if} d_P^{NIS^u}(x) < \left(d_P^{NIS^u}\right)^- \end{cases} \tag{5.9}$$

where $x_1^{u^*} = \left(x_{11}^{u^*}, x_{12}^{u^*}, \dots, x_{1n_1}^{u^*}\right)andx_2^{u^*} = \left(x_{21}^{u^*}, x_{22}^{u^*}, \dots, x_{2n_2}^{u^*}\right)$.

If $\alpha = min\left(\mu_1(x), \mu_2(x)\right)$, model (5.2) is equivalent to the form of Tchebycheff model (see [18,20]), which is equivalent to the following model:

$$\begin{aligned} & \max \alpha \\ & \text{subject to} \\ & \mu_1(x) \ge \alpha, \mu_2(x) \ge \alpha, \quad \alpha \in [0,1] \text{ and} \\ & x \in G = \left\{x = (x_1, x_2) \in R^n \middle| \tilde{A}_1 x_1 + \tilde{A}_2 x_2 \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x \ge 0, b \in R^m\right\} \ne \varphi, \end{aligned} \tag{5.10}$$

where α is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS. It is well known that if the optimal solution of (5.10) is the vector $(\alpha, \boldsymbol{x^{u^*}})$, then $\boldsymbol{x^{u^*}}$is the maximizing solution of model (5.2) and a satisfactory solution of the FLDMproblem.

As discussed previously, the basic concept of the bi-level programming technique is that the FLDM sets his goals and/or decisions with possible tolerances which are described by membership functions of fuzzy set theory. According to this concept, let t_k^Landt_k^R, $k = 1,2, \dots, n_1$ be the maximum acceptable negative and positive tolerance (relaxation) values on the decision vector considered by the FLDM, $x_1^{u^*} = \left(x_{11}^{u^*}, x_{12}^{u^*}, \dots, x_{1n_1}^{u^*}\right)$. The tolerances t_k^L and t_k^R are not necessarily the same. The tolerances give the second level decision maker an extent feasible region to search for the satisfactory solution. If the feasible region is empty, the negative and positive tolerances must be increased to give the second level decision maker an extent feasible region to search for the satisfactory solution [23, 28]. The linear membership functions (Fig.3) for each of the n_1 componentsof decision vector $x_1^{u^*} = \left(x_{11}^{u^*}, x_{12}^{u^*}, \dots, x_{1n_1}^{u^*}\right)$ controlled by the FLDM can be formulated as:

$$\mu_{x_{1k}}(x_{1k}) = \begin{cases} \frac{x_{1k}-\left(x_{1k}^{u^*}-t_k^L\right)}{t_k^L}, & \text{if } x_{1k}^{u^*}-t_k^L \le x_{1k} \le x_{1k}^{u^*} \\ \frac{\left(x_{1k}^{u^*}+t_k^R\right)-x_{1k}}{t_k^R}, & \text{if} x_{1k}^{u^*} \le x_{1k} \le x_{1k}^{u^*}+t_k^R,\ k=1,2,\dots,n_1 \\ 0, & \text{if otherwise} \end{cases} \tag{5.11}$$

It may be noted that, the decision maker may desire to shift the range of x_{1k}. Following Pramanik and Roy [23] and Sinha [28], this shift can be achieved.

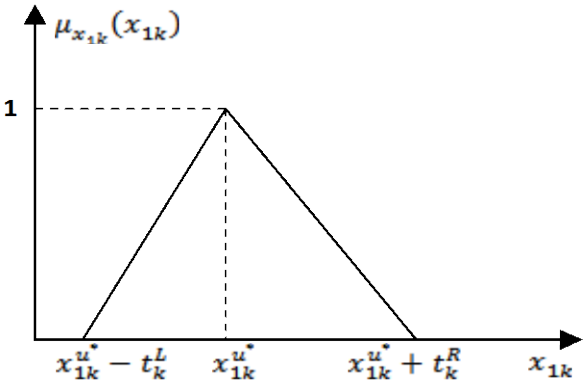

Fig.3: The membership function of the decision variable x_{1k}

5.2 The TOPSIS approach for BL-MOPP with fuzzy parameters

In order to obtain a compromise solution (satisfactory solution) to the BL-MOP problem with fuzzy parameters using the TOPSIS approach, the distance family of (2.2) to represent the distance function from the positive ideal solution,$d_P^{PIS^B}$, and the distance function from the negative ideal solution, $d_P^{NIS^B}$, can be proposed in this paper, for the objective functions of the first and second levels as follows:

$$d_P^{PIS^B}(x) = \left\{\sum_{j=1}^{m_1} w_{1j}^p \left[\frac{\tilde{f}_{1j}(x)-f_{1j}^*}{f_{1j}^- - f_{1j}^*}\right]^p + \sum_{j=1}^{m_2} w_{2j}^p \left[\frac{\tilde{f}_{2j}(x)-f_{2j}^*}{f_{2j}^- - f_{2j}^*}\right]^p\right\}^{\frac{1}{p}} \tag{5.12}$$

and

$$d_P^{NIS^B}(x) = \left\{\sum_{j=1}^{m_1} w_{1j}^p \left[\frac{f_{1j}^- - \tilde{f}_{1j}(x)}{f_{1j}^- - f_{1j}^*}\right]^p + \sum_{j=1}^{m_2} w_{2j}^p \left[\frac{f_{2j}^- - \tilde{f}_{2j}(x)}{f_{2j}^- - f_{2j}^*}\right]^p\right\}^{\frac{1}{p}} \tag{5.13}$$

where $w_k,\ k = 1,2,\dots,m_1+m_2$ are the relative importance (weights) of objectives in both levels.
$f_{ij}^* = \underbrace{min}_{x\in G} \tilde{f}_{ij}(x) = \underbrace{min}_{x\in G}\left(\tilde{f}_{ij}(x)\right)_\alpha^L, f_{ij}^- = \underbrace{max}_{x\in G} \tilde{f}_{ij}(x) = \underbrace{max}_{x\in G}\left(\tilde{f}_{ij}(x)\right)_\alpha^U,\ i=1,2,\ j=1,2,\dots,m_i,$ and $p = 1,2,\dots,\infty.$
Let $F^* = \left(f_{11}^*, f_{12}^*,\dots,f_{1m_1}^*, f_{21}^*, f_{22}^*,\dots,f_{2m_2}^*\right)$, the individual positive ideal solutions for both levels, and $F^- = \left(f_{11}^-, f_{12}^-,\dots,f_{1m_1}^-, f_{21}^-, f_{22}^-,\dots,f_{2m_2}^-\right)$, the individual negative ideal solutions for both levels. Similarly, for the special case of$p = \infty$, see [3,18] for the general form of the distance functions that can be applied to the proposed TOPSIS approach for solving BL-MOPP with fuzzy parameters.

In order to obtain a compromise solutions, we transfer the problem into the following bi-objective problem with two commensurable (but often conflicting) objectives [3-5, 18]:

$$\begin{aligned} &\text{Min } d_P^{PIS^B}(x) \\ &\text{Max } d_P^{NIS^B}(x) \\ &\text{subject to} \\ &x \in G = \left\{x = (x_1, x_2) \in R^n \middle| \tilde{A}_1 x_1 + \tilde{A}_2 x_2 \left(\begin{smallmatrix}\le\\=\\\ge\end{smallmatrix}\right) b, x \ge 0, b \in R^m\right\} \ne \varphi \end{aligned} \tag{5.14}$$

Since these two objectives are usually conflicting to each other, it is possible to simultaneously obtain their individual optima. Thus, we can use membership functions to represent these individual optima. Assume that the membership functions ($\mu_3(\boldsymbol{x})$and $\mu_4(\boldsymbol{x})$) of the two objective functions are linear between $\left(d_p^B\right)^*$ and $\left(d_p^B\right)^-$, they take the following form:

$$\left(d_P^{PIS^B}\right)^* = \underbrace{min}_{x\in G} d_P^{PIS^B}(x) \text{and the solution is } x^{PIS}, \tag{5.15}$$

$$\left(d_P^{NIS^B}\right)^* = \underbrace{max}_{x\in G} d_P^{NIS^B}(x) \text{and the solution is } x^{NIS}, \tag{5.16}$$

$$\left(d_P^{PIS^B}\right)^- = d_P^{PIS^B}(x^{NIS}) \text{or} \left(d_P^{PIS^B}\right)^- = \underbrace{max}_{x\in G} d_P^{PIS^B}(x) \text{ and} \tag{5.17}$$

$$\left(d_P^{NIS^B}\right)^- = d_P^{NIS^B}(x^{PIS}) \text{or } \left(d_P^{NIS^B}\right)^- = \underbrace{min}_{x\in G} d_P^{NIS^B}(x). \tag{5.18}$$

And also, assume that $\left(d_p^B\right)^* = \left(\left(d_P^{PIS^B}\right)^*, \left(d_P^{NIS^B}\right)^*\right)$ and $\left(d_p^B\right)^- = \left(\left(d_P^{PIS^B}\right)^-, \left(d_P^{NIS^B}\right)^-\right)$. Then, based on the preference concept, we assign a larger degree to the one with shorter distance from the PIS for $\mu_3(x) \equiv \mu_{d_P^{PIS^B}}(x)$ and assign a larger degree to the one with farther distance from NIS for $\mu_4(x) \equiv \mu_{d_P^{NIS^B}}(x)$. Therefore, as shown in Fig.2, $\mu_3(\boldsymbol{x})$ and $\mu_4(\boldsymbol{x})$ can be obtained as follows [3-5, 18]:

$$\mu_3(x) = \begin{cases} 1 & \text{if} d_P^{PIS^B}(x) < \left(d_P^{PIS^B}\right)^* \\ 1 - \dfrac{d_P^{PIS^B}(x) - \left(d_P^{PIS^B}\right)^*}{\left(d_P^{PIS^B}\right)^- - \left(d_P^{PIS^B}\right)^*} & \text{if } \left(d_P^{PIS^B}\right)^* \le d_P^{PIS^B}(x) \le \left(d_P^{PIS^B}\right)^- \\ 0 & \text{if} \left(d_P^{PIS^B}\right)^- < d_P^{PIS^B}(x) \end{cases} \tag{5.19}$$

$$\mu_4(x) = \begin{cases} 1 & \text{if} d_P^{NIS^B}(x) \ge \left(d_P^{NIS^B}\right)^* \\ 1 - \dfrac{\left(d_P^{NIS^B}\right)^* - d_P^{NIS^B}(x)}{\left(d_P^{NIS^B}\right)^* - \left(d_P^{NIS^B}\right)^-} & \text{if } \left(d_P^{NIS^B}\right)^- \le d_P^{NIS^B}(x) \le \left(d_P^{NIS^B}\right)^* \\ 0 & \text{if} d_P^{NIS^B}(x) < \left(d_P^{NIS^B}\right)^- \end{cases} \tag{5.20}$$

Applying the max-min decision model, which is proposed by Bellman and Zadeh [13] and extended by Zimmermann [35] the compromise solution $x^* = (x_1^*, x_2^*)$, of model (5.14) can be resolved and obtained by solving the following problem:

$$\mu_D(x) = \underbrace{max}_{x\in G} \left\{min\left(\mu_3(x), \mu_4(x)\right)\right\} \tag{5.21}$$

where $x_1^* = \left(x_{11}^*, x_{12}^*, \dots, x_{1n_1}^*\right)$ and $x_2^* = \left(x_{21}^*, x_{22}^*, \dots, x_{2n_2}^*\right)$. If $\delta = min\left(\mu_3(x), \mu_4(x)\right)$, the model (5.14) is equivalent to the form of Tchebycheff model [2-5,13, 26], which is equivalent to the following model:

$$\begin{aligned} &\max \delta \\ &\text{subject to} \\ &\mu_3(x) \ge \delta,\ \mu_4(x) \ge \delta \\ &x \in G = \left\{x = (x_1, x_2) \in R^n \,\middle|\, \tilde{A}_1 x_1 + \tilde{A}_2 x_2 \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x \ge 0, b \in R^m\right\} \ne \varphi, \\ &\text{and } \delta \in [0, 1]. \end{aligned} \tag{5.22}$$

where δ is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS. It is well known that if the optimal solution of (5.22) is the vector (δ, x_1^*, x_2^*), then $x^* = (x_1^*, x_2^*)$ is the maximizing solution of model (5.14).

Finally, as discussed in section 5.1, in order to generate the satisfactory solution of the BL-MOPP, $x^* = (x_1^*, x_2^*)$, the final proposed model that includes the membership function (5.11) for the first level decision variables vector, $x_1^{u^*} = \left(x_{11}^{u^*}, x_{12}^{u^*}, \dots, x_{1n_1}^{u^*}\right)$, is presented in[8] by Bakyas:

$$\begin{aligned} &\max \delta \\ &\text{subject to} \\ &1 - \frac{d_P^{PIS^B}(x) - \left(d_P^{PIS^B}\right)^*}{\left(d_P^{PIS^B}\right)^- - \left(d_P^{PIS^B}\right)^*} \ge \delta, \\ &1 - \frac{\left(d_P^{NIS^B}\right)^* - d_P^{NIS^B}(x)}{\left(d_P^{NIS^B}\right)^* - \left(d_P^{NIS^B}\right)^-} \ge \delta, \\ &\frac{x_{1k} - \left(x_{1k}^{u^*} - t_k^L\right)}{t_k^L} \ge \delta, \\ &\frac{\left(x_{1k}^{u^*} + t_k^R\right) - x_{1k}}{t_k^R} \ge \delta, \quad k = 1,2, \dots, n_1, \end{aligned}$$

$$x \in G = \left\{ x = (x_1, x_2) \in R^n \middle| \tilde{A}_1 x_1 + \tilde{A}_2 x_2 \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x \ge 0, b \in R^m \right\} \ne \varphi$$

and $\delta \in [0, 1]$. (5.23)

6. THE TOPSIS ALGORITHM FOR BL-MOPP WITH FUZZY PARAMETERS

The TOPSIS model (5.23) provides a satisfactory decision for the two DMs at the two levels. Following the above discussion, the algorithm for the proposed TOPSIS approach for solving BL-MOPP with fuzzy parameters is given as follows:

Step-1. Formulate the deterministic model of the BL-MOPP, for a prescribed value of α, i.e. the upper and lower bounds of their α-level are defined. Also, for the system constraints.

Step-2. Determine the individual maximum and minimum values for the upper and lower α-level of the objectives for both level DMs subject to the system constraints.

Step-3. Construct the PIS payoff table of the FLDM problem (5.1) from the lower bound model and obtain $F^{u^*} = \left(f_{11}^*, f_{12}^*, \dots, f_{1m_1}^*\right)$, the individual positive ideal solutions.

Step-4. Construct the NIS payoff table of the FLDM problem (5.1) from the upper bound model and obtain $F^{u^-} = \left(f_{11}^-, f_{12}^-, \dots, f_{1m_1}^-\right)$, the individual negative ideal solutions.

Step-5. Use Eq. (5.3) to construct $d_P^{PIS^u}(x)$ and $d_P^{NIS^u}(x)$.

Step-6. Ask the DM to select p, $\{p = 1,2, \dots, \infty\}$.

Step-7. Construct the payoff table of problem (5.2) and obtain $\left(d_p^u\right)^*$and $\left(d_p^u\right)^-$.

Step-8. Elicit the membership functions $\mu_{d_P^{PIS^u}}(x)$ and $\mu_{d_P^{NIS^u}}(x)$.

Step-9. Formulate the model (5.10) for the FLDM problem.

Step-10. Solve model (5.10) to get $x^{u^*} = \left(x_1^{u^*}, x_2^{u^*}\right)$, $x_1^{u^*} = \left(x_{11}^{u^*}, x_{12}^{u^*}, \dots, x_{1n_1}^{u^*}\right)$.

Step-11. Set the maximum negative and positive tolerance values on the decision vector $x_1^{u^*} = \left(x_{11}^{u^*}, x_{12}^{u^*}, \dots, x_{1n_1}^{u^*}\right)$, t_k^L and t_k^R, $k = 1,2, \dots, n_1$.

Step-12. Construct the PIS payoff table of the BL-MOPP from the lower bound model and obtain $F^* = \left(f_{11}^*, f_{12}^*, \dots, f_{1m_1}^* f_{21}^*, f_{22}^*, \dots, f_{2m_2}^*\right)$, the individual positive ideal solutions for both levels.

Step-13. Construct the NIS payoff table of the BL-MOPP from the upper bound model and obtain $F^- = \left(f_{11}^-, f_{12}^-, \dots, f_{1m_1}^- f_{21}^-, f_{22}^-, \dots, f_{2m_2}^-\right)$, the individual negative ideal solutions for both levels.

Step-14. Use Eqs. (5.12) and (5.13) to construct $d_P^{PIS^B}(x)$and$d_P^{NIS^B}(x)$, respectively.

Step-15. Construct the payoff table of problem (5.14) and obtain $\left(d_p^B\right)^*$and $\left(d_p^B\right)^-$.

Step-16. Elicit the membership functions $\mu_{d_P^{PIS^B}}(x)$ and $\mu_{d_P^{NIS^B}}(x)$.

Step-17. Elicit the membership functions $\mu_{x_{1k}}(x_{1k})$, $k = 1,2, \dots, n_1$.

Step-18. Formulate the model (5.23) for the BL-MOP problem.

Step-19. Solve model (5.23) to get$x^* = (x_1^*, x_2^*)$.

Step-20. If the DM is satisfied with the candidate solution in step 19, go to step 21, else go to step 22.

Step-21. Stop with a satisfactory solution, $x^* = (x_1^*, x_2^*)$, to the BL-MOPP with fuzzy parameters.

Step-22. Modify the maximum negative and positive tolerance values on the decision vector$x_1^{u^*} = \left(x_{11}^{u^*}, x_{12}^{u^*}, \dots, x_{1n_1}^{u^*}\right)$, t_k^L and t_k^R, $k = 1,2, \dots, n_1$, go to step 17.

7. A MODIFIED TOPSIS APPROACH FOR BL-MOPP WITH FUZZY PARAMETERS

In this section, a modified TOPSIS approach is presented in which, a fuzzy goal programming (FGP) approach, for more details see [9-11], is considered for solving the bi-criteria of the shortest distance from the PIS and the farthest distance from the NIS instead of the conventional max-min model. FGP is an extension of conventional goal programming (GP) introduced by Charnes and Cooper[37].The FGP approach to multi-objective decision making problems introduced by Mohamed[22] is extended by Baky[9,10] for solving DBL-MOP, multi-level multi-objective(ML-MOP) and BL-MOPP with fuzzy demands: FGP approach[40]. In decision making situation, the aim of each DM is to achieve highest membership value (unity) of the associated fuzzy goal in order to obtain the satisfactory solution. However, in real situation, achievement of all membership values to the highest degree (unity) is not possible due to conflicting objectives. Therefore, each DM should try to maximize his or her membership function by making them as close as possible to unity by minimizing their deviational variables. So for the defined membership functions in (5.7) and (5.8) of the FLDM problem the flexible membership goals having the aspired level unity can be represented as:

$$1 - \frac{d_P^{PIS^u}(x) - \left(d_P^{PIS^u}\right)^*}{\left(d_P^{PIS^u}\right)^- - \left(d_P^{PIS^u}\right)^*} + D_1^{PIS^-} - D_1^{PIS^+} = 1 \tag{7.1}$$

$$1 - \frac{\left(d_P^{NIS^u}\right)^* - d_P^{NIS^u}(x)}{\left(d_P^{NIS^u}\right)^* - \left(d_P^{NIS^u}\right)^-} + D_2^{NIS^-} - D_2^{NIS^+} = 1 \tag{7.2}$$

where$D_1^{PIS^-}, D_2^{NIS^-}$and$D_1^{PIS^+}, D_2^{NIS^+}$represent the under- andover-deviations from the aspired levels, respectively. The FGP approach of Mohamed [22] that solves single-level multi-objective linear programming problem is considered to solve the TOPSIS model of the FLDM problem (5.2) as follows:

$$\begin{aligned}
&\min\ Z = w_1^{PIS} D_1^{PIS^+} + w_2^{NIS} D_2^{NIS^-}\\
&\text{subject to}\\
&1 - \frac{d_P^{PIS^u}(x) - \left(d_P^{PIS^u}\right)^*}{\left(d_P^{PIS^u}\right)^- - \left(d_P^{PIS^u}\right)^*} + D_1^{PIS^-} - D_1^{PIS^+} = 1\\
&1 - \frac{\left(d_P^{NIS^u}\right)^* - d_P^{NIS^u}(x)}{\left(d_P^{NIS^u}\right)^* - \left(d_P^{NIS^u}\right)^-} + D_2^{NIS^-} - D_2^{NIS^+} = 1\\
&x \in G = \left\{ x = (x_1, x_2) \in R^n \,\middle|\, \tilde{A}_1 x_1 + \tilde{A}_2 x_2 \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x \ge 0, b \in R^m \right\} \ne \varphi
\end{aligned} \tag{7.3}$$

where the numerical weights w_1^{PIS} and w_2^{NIS}represent the relative importance of achieving the aspired levels of the respective fuzzy goals subject to the constraints set in the decision situation. The weighting scheme suggested by Mohamed [22] can be used to assign the values of w_1^{PIS} and w_2^{NIS} as follows:

$$w_1^{PIS} = \frac{1}{\left(d_P^{PIS^u}\right)^- - \left(d_P^{PIS^u}\right)^*} \text{and} w_2^{NIS} = \frac{1}{\left(d_P^{NIS^u}\right)^* - \left(d_P^{NIS^u}\right)^-} \tag{7.4}$$

As discussed briefly in section 5, the FLDM sets his goals and/or decisions with possible tolerances which are described by membership functions of fuzzy set theory. Then the BL-MOP problem with fuzzy parameters is reduced to the TOPSIS model (5.14). In order to obtain a satisfactory solution, a FGP model for solving the bi-criteria problem (5.14) is formulated as:

$$\begin{aligned}
&\min\ Z = w_3^{PIS} D_3^{PIS^+} + w_4^{NIS} D_4^{NIS^-} + \sum_{k=1}^{n_1} \left[w_{1k}^L \left(D_{1k}^{L-} + D_{1k}^{L+}\right) + w_{1k}^R \left(D_{1k}^{R-} + D_{1k}^{R+}\right) \right]\\
&\text{subject to}\\
&1 - \frac{d_P^{PIS^B}(x) - \left(d_P^{PIS^B}\right)^*}{\left(d_P^{PIS^B}\right)^- - \left(d_P^{PIS^B}\right)^*} + D_3^{PIS^-} - D_3^{PIS^+} = 1\\
&1 - \frac{\left(d_P^{NIS^B}\right)^* - d_P^{NIS^B}(x)}{\left(d_P^{NIS^B}\right)^* - \left(d_P^{NIS^B}\right)^-} + D_4^{NIS^-} - D_4^{NIS^+} = 1\\
&\frac{x_{1k} - \left(x_{1k}^{u^*} - t_k^L\right)}{t_k^L} + D_{1k}^{L-} - D_{1k}^{L+} = 1, \quad k = 1,2 \dots, n_1.\\
&\frac{\left(x_{1k}^{u^*} + t_k^R\right) - x_{1k}}{t_k^R} + D_{1k}^{R-} - D_{1k}^{R+} = 1, \quad k = 1,2 \dots, n_1.
\end{aligned} \tag{7.5}$$

$$x \in G = \left\{ x = (x_1, x_2) \in R^n \middle| \tilde{A}_1 x_1 + \tilde{A}_2 x_2 \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x \ge 0, b \in R^m \right\} \ne \varphi$$

where $D_{1k}^- = (D_{1k}^{L-}, D_{1k}^{R-})$, $D_{1k}^+ = (D_{1k}^{L+}, D_{1k}^{R+})$ and $D_3^{PIS^-}, D_4^{NIS^-}, D_{1k}^{L-}, D_{1k}^{R-}, D_3^{PIS^+}, D_4^{NIS^+}, D_{1k}^{L+}, D_{1k}^{R+} \ge 0$ with $D_3^{PIS^-} \times D_3^{PIS^+} = 0, D_4^{NIS^-} \times D_4^{NIS^+} = 0, D_{1k}^{L-} \times D_{1k}^{L+} = 0$ and $D_{1k}^{R-} \times D_{1k}^{R+} = 0$, $k = 1,2,\ldots,n_1$, represent the under- and over-deviation, respectively, from the aspired levels. Also Z represents the fuzzy achievement function. Again, to assess the relative importance of the fuzzy goals properly, the weighting scheme suggested by Mohamed [22] can be used to assign the values to $w_3^{PIS}, w_4^{NIS}, w_{1k}^L$ and w_{1k}^R as:

$$w_3^{PIS} = \frac{1}{\left(d_P^{PIS^B}\right)^- - \left(d_P^{PIS^B}\right)^*} \text{ and } w_4^{NIS} = \frac{1}{\left(d_P^{NIS^B}\right)^* - \left(d_P^{NIS^B}\right)^-} \tag{7.6}$$

$$w_{1k}^L = \frac{1}{t_k^L} \text{ and } w_{1k}^R = \frac{1}{t_k^R}, k = 1,2\ldots,n_1. \tag{7.7}$$

8. ILLUSTRATIVE NUMERICAL EXAMPLE

The following numerical example studied by Pramanik and Dey [24] is considered to illustrate the proposed TOPSIS and modified TOPSIS approaches for solving BL-MOPP with fuzzy parameters.

[1st level]

$$\underbrace{\text{Min}}_{x_1,x_2} \begin{pmatrix} \tilde{f}_{11}(x) = (x_1 + \tilde{3}x_2 + \tilde{2}x_3 + \tilde{3}x_4), \\ \tilde{f}_{12}(x) = (\tilde{2}x_1 + \tilde{9}x_2 + \tilde{3}x_3 + \tilde{5}x_4), \\ \tilde{f}_{13}(x) = (\tilde{3}x_1 + \tilde{9}x_2 + \tilde{9}x_3 + x_4) \end{pmatrix}$$

[2nd level]

$$\underbrace{\text{Min}}_{x_3,x_4} \begin{pmatrix} \tilde{f}_{21}(x) = (\tilde{6}x_1 + \tilde{3}x_2 + \tilde{2}x_3 + \tilde{2}x_4), \\ \tilde{f}_{22}(x) = (\tilde{5}x_1 + \tilde{9}x_2 - \tilde{9}x_3 + \tilde{6}x_4) \end{pmatrix}$$

subject to

$\tilde{3}x_1 - x_2 + x_3 + \tilde{3}x_4 \le \widetilde{48}, \tilde{2}x_1 + \tilde{4}x_2 + \tilde{2}x_3 - \tilde{2}x_4 \le \widetilde{35},$

$x_1 + \tilde{2}x_2 - x_3 + x_4 \ge \widetilde{30}, \quad x_1, x_2, x_3, x_4 \ge 0.$

Here, the fuzzy numbers are assumed to be triangular fuzzy numbers and are given as follows:

$\tilde{2} = (0,2,3)$, $\tilde{3} = (2,3,4)$, $\tilde{4} = (3,4,5)$, $\tilde{5} = (4,5,6)$, $\tilde{6} = (5,6,7)$, $\tilde{8} = (6,8,10)$, $\tilde{9} = (8,9,10)$, $\widetilde{30} = (28,30,32)$. $\widetilde{35} = (33,35,37)$, $\widetilde{48} = (45,48,49)$.

Since the problem is minimization-type then replacing the fuzzy coefficient by their lower bound α-cuts, the above problem can be written as:

[1st level]

$$\underbrace{\text{Min}}_{x_1,x_2} \begin{pmatrix} \left(\tilde{f}_{11}(x)\right)_\alpha^L = x_1 + (2+\alpha)x_2 + (2\alpha)x_3 + (2+\alpha)x_4 \\ \left(\tilde{f}_{12}(x)\right)_\alpha^L = (2\alpha)x_1 + (8+\alpha)x_2 + (2+\alpha)x_3 + (4+\alpha)x_4 \\ \left(\tilde{f}_{13}(x)\right)_\alpha^L = (2+\alpha)x_1 + (8+\alpha)x_2 + (8+\alpha)x_3 + x_4 \end{pmatrix}$$

[2nd level]

$$\underbrace{\text{Min}}_{x_3,x_4} \begin{pmatrix} \left(\tilde{f}_{21}(x)\right)_\alpha^L = (5+\alpha)x_1 + (2+\alpha)x_2 + (2\alpha)x_3 + (2\alpha)x_4 \\ \left(\tilde{f}_{22}(x)\right)_\alpha^L = (4+\alpha)x_1 + (8+\alpha)x_2 - (8+\alpha)x_3 + (5+\alpha)x_4 \end{pmatrix}$$

subject to

$(2+\alpha)x_1 - x_2 + x_3 + (2+\alpha)x_4 \le 49 - \alpha,$

$(2\alpha)x_1 + (3+\alpha)x_2 + (2\alpha)x_3 - (2\alpha)x_4 \le 37 - 2\alpha,$

$x_1 + (3-\alpha)x_2 - x_3 + x_4 \ge 28 + 2\alpha,$

$x_1, x_2, x_3, x_4 \ge 0.$

For, α=0.5, the BL-MOPP with fuzzy parameters reduces to a deterministic BL-MOP problem as follows:

[1st level]

$$\underbrace{\text{Min}}_{x_1,x_2}\begin{pmatrix} \left(\tilde{f}_{11}(x)\right)^{L}_{0.5} = x_1 + 2.5x_2 + x_3 + 2.5x_4 \\ \left(\tilde{f}_{12}(x)\right)^{L}_{0.5} = x_1 + 8.5x_2 + 2.5x_3 + 4.5x_4 \\ \left(\tilde{f}_{13}(x)\right)^{L}_{0.5} = 2.5x_1 + 8.5x_2 + 8.5x_3 + x_4 \end{pmatrix}$$

[2nd level]

$$\underbrace{\text{Min}}_{x_3,x_4}\begin{pmatrix} \left(\tilde{f}_{21}(x)\right)^{L}_{0.5} = 5.5x_1 + 2.5x_2 + x_3 + x_4 \\ \left(\tilde{f}_{22}(x)\right)^{L}_{0.5} = 4.5x_1 + 8.5x_2 - 8.5x_3 + 5.5x_4 \end{pmatrix}$$

subject to

$$\boldsymbol{x} \in G = \left\{\boldsymbol{x} = (x_1, x_2, x_3, x_4) \middle| \begin{matrix} 2.5x_1 - x_2 + x_3 + 2.5x_4 \le 48.5 \quad x_1 + 2.5x_2 - x_3 + x_4 \ge 29 \\ x_1 + 3.5x_2 + x_3 - x_4 \le 36 x_1, x_2, x_3, x_4 \ge 0. \end{matrix}\right\}$$

The individual minimum and maximum of each of the objective functions at both levels calculated from the lower and upperbound α-cut model are shown in Table 1.

Table-1: minimumand maximum individual optimal solutions.

Objective Function	$\left(\tilde{f}_{11}(x)\right)^{L,U}_{0.5}$	$\left(\tilde{f}_{12}(x)\right)^{L,U}_{0.5}$	$\left(\tilde{f}_{13}(x)\right)^{L,U}_{0.5}$	$\left(\tilde{f}_{21}(x)\right)^{L,U}_{0.5}$	$\left(\tilde{f}_{22}(x)\right)^{L,U}_{0.5}$
$min\left(\tilde{f}_{ij}(x)\right)^{L}_{0.5}$	29	48.862	48.862	29	55.875
$max\left(\tilde{f}_{ij}(x)\right)^{U}_{0.5}$	155.47	315.79	268.46	152.1	342.34

8.1 TOPSIS approach Solution:

First-level MOP problem:

We first obtain PIS and NIS payoff tables for the FLMD problem from the lower and upper bound model respectively (Table 2 and 3):

Table-2: PIS payoff table of the FLDM problem

Objective Function	$\left(\tilde{f}_{11}(x)\right)^{L}_{0.5}$	$\left(\tilde{f}_{12}(x)\right)^{L}_{0.5}$	$\left(\tilde{f}_{13}(x)\right)^{L}_{0.5}$	x_1	x_2	x_3	x_4
$min\left(\tilde{f}_{11}(x)\right)^{L}_{0.5}$	29^*	71	88.25	11.5	7	0	0
$min\left(\tilde{f}_{12}(x)\right)^{L}_{0.5}$	29.005	48.862^*	79.96	20.73	3.31	0	0
$min\left(\tilde{f}_{13}(x)\right)^{L}_{0.5}$	60.1	121.4	48.862^*	0	3.31	0	20.73

$F^{u^*} = (f^*_{11}, f^*_{12}, f^*_{13}) = (29, 48.862, 48.862)$

Table-3: NIS payoff table of the FLDM problem

Objective Function	$\left(\tilde{f}_{11}(x)\right)^{U}_{0.5}$	$\left(\tilde{f}_{12}(x)\right)^{U}_{0.5}$	$\left(\tilde{f}_{13}(x)\right)^{U}_{0.5}$	x_1	x_2	x_3	x_4
$max\left(\tilde{f}_{11}(x)\right)^{U}_{0.5}$	155.47^*	315.79	196.32	0	17.87	0	26.55
$max\left(\tilde{f}_{12}(x)\right)^{U}_{0.5}$	155.47	315.79^*	196.32	0	17.87	0	26.55
$max\left(\tilde{f}_{13}(x)\right)^{U}_{0.5}$	138.11	253.67	268.46^*	0	10.38	15.85	17.5

$F^{u^-} = (f^-_{11}, f^-_{12}, f^-_{13}) = (155.47, 315.79, 268.46)$

Assume that $w_i = \frac{1}{3}$ $i = 1, 2, 3$, the equations for $d_P^{PIS^u}(x)$ and $d_P^{NIS^u}(x)$ when $p = 2$ are:

$$F_1^u = d_2^{PIS^u}(x) = \begin{Bmatrix} 0.0000069[(x_1 + 2.5x_2 + x_3 + 2.5x_4) - 29]^2 + \\ 0.00000156[(x_1 + 8.5x_2 + 2.5x_3 + 4.5x_4) - 48.862]^2 + \\ 0.0000023[(2.5x_1 + 8.5x_2 + 8.5x_3 + x_4) - 48.862]^2 \end{Bmatrix}^{1/2}$$

$$F_2^u = d_2^{NIS^u}(x) = \begin{Bmatrix} 0.0000069[155.47 - (x_1 + 2.5x_2 + x_3 + 2.5x_4)]^2 + \\ 0.00000156[315.79 - (x_1 + 8.5x_2 + 2.5x_3 + 4.5x_4)]^2 + \\ 0.0000023[268.46 - (2.5x_1 + 8.5x_2 + 8.5x_3 + x_4)]^2 \end{Bmatrix}^{1/2}$$

Next to formulate model (5.10) we determine the following $max\ F_1^u = 0.41$, $min\ F_1^u = 0.044$, and $max\ F_2^u = 0.553$, $min\ F_2^u = 0.188$, as proposed in this paper. Thus we have $d_2^{u^*} = (0.044, 0.553)$ and $d_2^{u^-} = (0.41, 0.188)$, therefore, the membership functions $\mu_1(x)$ *and* $\mu_2(x)$ can be obtained as:

$$\mu_{F_1^u}(x) = 1.12 - 2.73F_1^u$$
$$\mu_{F_2^u}(x) = -0.515 + 2.74F_2^u$$

And then, the equivalent TOPSIS formulation for the FLDM problem is obtained as:

$$\begin{aligned} &\max \alpha \\ &\text{subject to} \\ &1.12 - 2.73F_1^u \geq \alpha \\ &-0.515 + 2.74F_2^u \geq \alpha \\ &2.5x_1 - x_2 + x_3 + 2.5x_4 \leq 48.5 \\ &x_1 + 2.5x_2 - x_3 + x_4 \geq 29 \\ &x_1 + 3.5x_2 + x_3 - x_4 \leq 36 \\ &x_1, x_2, x_3, x_4 \geq 0. \ \ and \ \ \alpha \in [0,1] \end{aligned}$$

The maximum satisfactory level $\alpha = 0.9946$ is achieved for the solution $x^{u^*} = (20.13, 3.31, 0, 0.595)$. Let the first level DM decide $x_1^{u^*} = 20.13, x_2^{u^*} = 3.31$, with positive tolerance $t_1^R = t_2^R = 0.5$ (one sided membership function [23,28]).

The BL-MOP problem:

We first obtain PIS and NIS payoff tables for the second level MOP problem from the lower and upper bound model respectively (Tables 4 and 5):

Table-4: PIS payoff table of the SLDM problem

Objective Function	$\left(\tilde{f}_{21}(x)\right)_{0.5}^{L}$	$\left(\tilde{f}_{22}(x)\right)_{0.5}^{L}$	x_1	x_2	x_3	x_4
$min\left(\tilde{f}_{21}(x)\right)_{0.5}^{L}$	29^*	102.6	0	10.83	0	1.917
$min\left(\tilde{f}_{22}(x)\right)_{0.5}^{L}$	60.16	55.875^*	0	10.83	15.58	17.5

$F^{l^*} = (f_{21}^*, f_{22}^*) = (29, 55.875)$

Table-5: NIS payoff table of the SLDM problem

Objective Function	$\left(\tilde{f}_{21}(x)\right)_{0.5}^{U}$	$\left(\tilde{f}_{22}(x)\right)_{0.5}^{U}$	x_1	x_2	x_3	x_4
$min\left(\tilde{f}_{21}(x)\right)_{0.5}^{U}$	152.1^*	156.52	21.1	4.26	0	0
$min\left(\tilde{f}_{22}(x)\right)_{0.5}^{U}$	128.92	342.34^*	0	17.87	0	26.55

$F^{l^-} = (f_{21}^-, f_{22}^-) = (152.1, 342.34)$

Assume that $w_i = \frac{1}{5}\ \ i = 1, 2, 3, 4, 5$, the equations for $d_P^{PIS^B}(x)$ and $d_P^{NIS^B}(x)$ when $p = 2$ are:

$$P_1^B = d_2^{PIS^B}(x) = \begin{Bmatrix} 0.0000025[(x_1 + 2.5x_2 + x_3 + 2.5x_4) - 29]^2 + \\ 0.00000056\ [(x_1 + 8.5x_2 + 2.5x_3 + 4.5x_4) - 48.862]^2 + \\ 0.00000083[(2.5x_1 + 8.5x_2 + 8.5x_3 + x_4) - 48.862]^2 + \\ 0.0000026[(5.5x_1 + 2.5x_2 + x_3 + x_4) - 29]^2 + \\ 0.00000049[(4.5x_1 + 8.5x_2 - 8.5x_3 + 5.5x_4) - 55.875]^2 \end{Bmatrix}^{1/2}$$

$$P_2^B = d_2^{NIS^B}(x) = \begin{Bmatrix} 0.0000025[155.47 - (x_1 + 2.5x_2 + x_3 + 2.5x_4)]^2 + \\ 0.00000056\ [315.79 - (x_1 + 8.5x_2 + 2.5x_3 + 4.5x_4)]^2 + \\ 0.00000083[268.46 - (2.5x_1 + 8.5x_2 + 8.5x_3 + x_4)]^2 + \\ 0.0000026[152.1 - (5.5x_1 + 2.5x_2 + x_3 + x_4)]^2 + \\ 0.00000049[342.34 - (4.5x_1 + 8.5x_2 - 8.5x_3 + 5.5x_4)]^2 \end{Bmatrix}^{1/2}$$

Next to formulate model (5.23) we determine the following $max\, P_1^B = 0.303$, $min\, P_1^B = 0.065$, and $max\, P_2^B = 0.396$, $min\, P_2^B = 0.175$, as proposed in this paper. Thus we have $d_2^{B^*} = (0.065, 0.396)$and $d_2^{B^-} = (0.303,0.175\,)$, therefore, the membership functions $\mu_3(x)\; and\; \mu_4(x)$ can be obtained as:

$$\mu_{P_1^B}(x) = 1.27 - 4.23P_1^B$$
$$\mu_{P_2^B}(x) = -0.792 + 4.52P_2^B$$

Finally, the equivalent TOPSIS formulation for the BL-MOPP is obtained as:

$$\begin{aligned}
&\max \delta \\
&\text{subject to} \\
&1.27 - 4.23P_1^B \geq \delta \\
&-0.792 + 4.52P_2^B \geq \delta \\
&41.26 - 2x_1 \geq \delta \\
&7.62 - 2x_2 \geq \delta \\
&2.5x_1 - x_2 + x_3 + 2.5x_4 \leq 48.5 \\
&x_1 + 2.5x_2 - x_3 + x_4 \geq 29 \\
&x_1 + 3.5x_2 + x_3 - x_4 \leq 36 \\
&x_1, x_2, x_3, x_4 \geq 0, \quad and \quad \alpha \in [0, 1]
\end{aligned}$$

The maximum overall satisfactory level of the BL-MOP problem $\delta = 0.8997$ is achieved for the solution $x^* = (1.812, 3.36, 0, 18.79)$, with objective function values $f_{11} = 57.19$, $f_{12} = 114.93$, $f_{13} = 51.88$, $f_{21} = 37.16$, and $f_{22} = 140.1$, and with membership function values $\mu_{11} = 0.78$, $\mu_{12} = 0.75$, $\mu_{13} = 0.986$, $\mu_{21} = 0.94$, and $\mu_{22} = 0.71$, respectively.

Table-6: Comparison between the numerical results of the TOPSIS approach and the method of Pramanik and Dey [24].

The TOPSIS approach		The method of Pramanik and Dey		The optimal solution
$f_{11} = 57.19$	$\mu_{11} = 0.78$	$f_{11} = 37$	$\mu_{11} = 0.902$	$f_{11} = 29$
$f_{12} = 114.93$	$\mu_{12} = 0.75$	$f_{12} = 90$	$\mu_{12} = 0.815$	$f_{12} = 48.862$
$f_{13} = 51.88$	$\mu_{13} = 0.986$	$f_{13} = 108.25$	$\mu_{13} = 0.692$	$f_{13} = 48.862$
$f_{21} = 37.16$	$\mu_{21} = 0.94$	$f_{21} = 78.25$	$\mu_{21} = 0.496$	$f_{21} = 29$
$f_{22} = 140.1$	$\mu_{22} = 0.71$	$f_{22} = 105.5$	$\mu_{22} = 0.795$	$f_{22} = 55.875$

A comparison given in Table 6 between the TOPSIS approach and the method given in [24] by Pramanik and Dey shows that themethod of Pramanik and Dey nearly preferred than that of the first method.

8.2 Modified TOPSIS approach Solution

Following the discussion of section 7, and considering the values in Table 1 that summarizes minimum and maximum individual optimal solutions. The proposed modified TOPSIS procedure to the BL-MOPP with fuzzy parameters proceeds as:

<u>The modified TOPSIS model of the FLDM:</u>

$$\begin{aligned}
&\min\ Z = 2.73\, D_1^{PIS^+} + 2.74 D_2^{NIS^-} \\
&\text{subject to} \\
&1.12 - 2.73F_1^u + D_1^{PIS^-} - D_1^{PIS^+} = 1 \\
&-0.515 + 2.74F_2^u + D_2^{NIS^-} - D_2^{NIS^+} = 1 \\
&2.5x_1 - x_2 + x_3 + 2.5x_4 \leq 48.5 \\
&x_1 + 2.5x_2 - x_3 + x_4 \geq 29 \\
&x_1 + 3.5x_2 + x_3 - x_4 \leq 36 \\
&x_1, x_2, x_3, x_4 \geq 0. \quad D_1^{PIS^-}, \quad D_1^{PIS^+}, \quad D_2^{NIS^-}, \quad D_2^{NIS^+} \geq 0,
\end{aligned}$$

The optimal solution of the FLDM problem is achieved at $x^{u^*} = (20.678, 3.315, 0.0096, 0.044)$. Let the first level DM decide $x_1^{u^*} = 20.678$, and $x_2^{u^*} = 3.315$, with positive tolerances $t_1^R = t_2^R = 0.5$ and weights of $w_{1k}^R = \frac{1}{0.5} = 2$, (one sided membership function [23,28]).

The modified TOPSIS model of the BL-MOP problem:

$$\min\ Z = 4.23D_3^{PIS^+} + 4.52D_4^{NIS^-} + 2D_1^{R-} + 2D_1^{R+} + 2D_2^{R-} + 2D_2^{R+}$$

subject to

$$1.27 - 4.23P_1^B + D_3^{PIS^-} - D_3^{PIS^+} = 1$$
$$-0.792 + 4.52P_2^B + D_4^{NIS^-} - D_4^{NIS^+} = 1$$
$$42.356 - 2x_1 + D_1^{R-} - D_1^{R+} = 1$$
$$7.64 - 2x_2 + D_2^{R-} - D_2^{R+} = 1$$
$$2.5x_1 - x_2 + x_3 + 2.5x_4 \le 48.5$$
$$x_1 + 2.5x_2 - x_3 + x_4 \ge 29$$
$$x_1 + 3.5x_2 + x_3 - x_4 \le 36$$
$$x_1, x_2, x_3, x_4 \ge 0. \quad D_1^{PIS^-}, \quad D_1^{PIS^+}, \quad D_2^{NIS^-}, \quad D_2^{NIS^+} \ge 0,$$
$$D_3^{PIS^-} \times D_3^{PIS^+} = 0,\ D_4^{NIS^-} \times D_4^{NIS^+} = 0, \text{ and } D_k^{R-} \times D_k^{R+} = 0 \quad k = 1,2$$

The satisfactory solution of the BL-MOPP with fuzzy parameters is $x^* = (20.68, 3.32, 0\ , 0.02)$ with objective function values $f_{11} = 29.03,\ f_{12} = 48.99,\ f_{13} = 79.94,\ f_{21} = 122.1$,and $f_{22} = 121.4$, and with membership function values $\mu_{11} = 0.9997,\ \mu_{12} = 0.9995,\ \mu_{13} = 0.858,\ \mu_{21} = 0.244$, and $\mu_{22} = 0.771$, respectively.

A comparison given in Table 7 between the modified TOPSIS method and that given in [24] by Pramanik and Dey shows that the compromise solution of the modified TOPSIS method, in this paper, is great preferred than that given in [24] by Pramanik and Dey.

Table-7: Comparison between the numerical results of the modified TOPSIS approach and the method of Pramanik and Dey [24].

The modified TOPSIS approach		The method of Pramanik and Dey		The optimal solution
$f_{11} = 29.03$	$\mu_{11} = 0.9997$	$f_{11} = 37$	$\mu_{11} = 0.902$	$f_{11} = 29$
$f_{12} = 48.99$	$\mu_{12} = 0.9995$	$f_{12} = 90$	$\mu_{12} = 0.815$	$f_{12} = 48.862$
$f_{13} = 79.94$	$\mu_{13} = 0.858$	$f_{13} = 108.25$	$\mu_{13} = 0.692$	$f_{13} = 48.862$
$f_{21} = 122.1$	$\mu_{21} = 0.244$	$f_{21} = 78.25$	$\mu_{21} = 0.496$	$f_{21} = 29$
$f_{22} = 121.4$	$\mu_{22} = 0.771$	$f_{22} = 105.5$	$\mu_{22} = 0.795$	$f_{22} = 55.875$

For, indicating the merits of the modified TOPSIS approach. A comparison given in Table 8 between the TOPSIS approach, the modified TOPSIS approach and the FGP algorithm (40) by Baky *et al.* for solving the BL-MOPP with fuzzy parameters shows that the values of objective functions and membership functions for the BL-MOPP with fuzzy parameters obtained from the modified TOPSIS is more preferred than that given by the TOPSIS methods and FGP algorithm. As the modified TOPSIS approach combines the advantages of TOPSIS approach and FGP approach. As the TOPSIS approach transfers q objectives which are conflicting and non-commensurable into two objectives (the shortest distance from the PIS and the longest distance from the NIS), which are commensurable and most of time conflicting. And FGP approach solves the bi-objective problem to obtain the satisfactory solution.

Table-8: Comparison between the numerical results of the modified TOPSIS, TOPSIS approach and FGP algorithm by Baky *et al.*[40].

The modified TOPSIS approach	The TOPSIS approach	The FGP algorithm
$f_{11} = 29.03\mu_{11} = 0.9997$	$f_{11} = 57.19\mu_{11} = 0.78$	$f_{11} = 29\mu_{11} = 1$
$f_{12} = 48.99\mu_{12} = 0.9995$	$f_{12} = 114.93\mu_{12} = 0.75$	$f_{12} = 70.95\mu_{12} = 0.917$
$f_{13} = 79.94\mu_{13} = 0.858$	$f_{13} = 51.88\mu_{13} = 0.986$	$f_{13} = 88.27\mu_{13} = 0.821$
$f_{21} = 122.1\mu_{21} = 0.244$	$f_{21} = 37.16\mu_{21} = 0.94$	$f_{21} = 80.9\mu_{21} = 0.578$
$f_{22} = 121.4\mu_{22} = 0.771$	$f_{22} = 140.1\mu_{22} = 0.71$	$f_{22} = 111.27\mu_{22} = 0.81$

9. CONCLUSION AND SUMMARY

Considering the advantage of the TOPSIS approach for MODM, this paper proposes a TOPSIS approach and modified TOPSIS approach for solving BL-MOPP with fuzzy parameters. In order to obtain a compromise (satisfactory) solution to the BL-MOPP with fuzzy parameters using the TOPSIS or modified TOPSIS approach, the distance function from the positive ideal solution and the distance function from the negative ideal solution, in the proposed formulation in this paper, the objective functions of both the upper and lower levels. Then, the bi-objective problem can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS compromise solution by a second-order compromise. In the TOPSIS approach, the max-min operator is then considered as a suitable one to resolve the conflict between the new criteria (the shortest distance from the PIS and the longest distance from the NIS). Also, in the modified TOPSIS approach, the FGP approach is considered for solving the conflict between the new criteria. An illustrative numerical example is given to demonstrate the proposed TOPSIS and modified TOPSIS approach for BL-MOPP with fuzzy parameters. A comparison between the proposed TOPSIS approach, modified TOPSIS approach and the FGP algorithm proposed in by Baky *et al.* [40] given in Table 8, shows that the satisfactory solution of the modified TOPSIS approach is more preferred than the satisfactory solution of the others. However, the authors hope that the approach presented in this article will open up new possibilities of research on modified TOPSIS approach for dealing with multi-level multi-objective programming problems with fuzzy parameters for its practical implementation to real world hierarchical decision making problems.

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